CSIR-NET Full length TEST PAPER

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ATS-1

(Full Length-Q.M. & MP.)

PART-A

1. If two numbers are given, each number multiplied by sum of both numbers,. Then the multiplication is 247 and 114 respectively. Find the sum of these two numbers.

(a)19 (b)20

(c)21 (d)23

2. How many numbers between 6 and 1300 are completely divisible by 6 or 9 or by both?

(a) 297 (b)293

(c)287 (d)288

3. The sum of n terms of an A.P is $3n^2 + 5n$ then 164 is which digit?

(a) 24th (b) 27th

(c) 26th (d) 25th

4. If p : q = 7 :9 and q : r = 15 :7 then p : r is-

(a) 3 : 5 (b) 5 : 3

(c) 7 : 15 (d) 7 : 21

5. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between their interior angles is 2 : 3. The number of sides of these polygons is respectively-

(a) 6,12 (b) 5,10

(c)4,8 (d)7,14

6. A daily sheet calendar of the year 2013 contains sheets of 10 × 10 cm size. All the sheets of the calendar are spread over the floor of a room of 5m × 7.3m size. What percentage of the floor will be covered by these sheets?

(a) 0.1 (b)1

(c)10 (d)100

7. There are 2 hills, A and B, in a region. If hill A is located N30ºE of hill B, what will be the direction of hill B when observed from hill A? (N 30ºE means 30º from north towards east).

(a) S 30ºW (b) S 60ºW

(c) S 30ºE (d) S 60ºE

8. 20% of students of a particular course get jobs within one year of passing. 20% of the remaining students get jobs by the end of second year of passing. If 16 students are still jobless, how many students had passed the course?

(a) 32 (b)64

(c)25 (d)100

9. Three identical flat equilateral-triangular plates of side 5 cm each are placed together such that they form a trapezium. The length of the longer of the two parallel sides of this trapezium is

(a) 5 √3/4 cm (b) 5 √2 cm

(c) 10 cm (d) 10 √3 cm

10. A code consists of at most two identical letters followed by at most four identical digits. The code must have at least one letter and one digit. How many distinct codes can be generated using letters A to Z and digits 1 to 9?

(a) 936 (b) 1148

(c)1872 (d)2511

11. A series is given with one term missing. Select the correct alternative from the given ones that will complete the series. GH, LM, PQ, ST, ?

(a)UZ (b)UX

(c)UY (d)UV

12. In the following question, select the missing number from the given series.

9, 18, 72, 576, ?

(a)8116 (b)8216

(c)9016 (d)9216

13. Which one set of letters when sequentially placed at gap in the given letter series shall complete it.__nmmn__mmnn__mnnm__

(a) nnmm (b) nmmn

(c) mnmn (d) mmnm

14. Direction : In each of the following questions, select the related word/letters/number from the given alternatives. VISH : ERHS :: NAME : ?

(a) MZME (b) MZNE

(c) MNZE (d) MZNV

15. Direction : In each of the following questions, select the one which is different from the others.

(A) (12-144) (B) (13-156) (C) (14-166) (D) (15-180)

(a)A (b)B

(c)C (d)D

16. A solid contains a spherical cavity. The cavity is filled with a liquid and includes a spherical bubble of gas. The radii of cavity and gas bubble are 2 mm and I mm, respectively. What proportion of the cavity is filled with liquid ?

(a)18 (b)38

(c)58 (d)78

17. Which one of the following statements is logically incorrect ?

(a) I always speak the truth (b) I occasionally lie

(c) I occasionally speak the truth (d) I always lie

18. N is a four digit number. If the leftmost digit is removed, the resulting three digit number is 1/9th Of N. How many such N are possible ?

(a)10 (b)9

(c)8 (d)7

19. "My friend Raju has more than 1000 books". said Ram. "Oh no, he has less than 1000 book said Shyam. "Well. Raju certainly has at least one book", said Geeta. If only one of these statements is true. how many books does Raju have ?

(a)1 (b)999

(c)1000 (d)1001

20. Fill in the blank : F2, ..., D8, C16, B32, A64.

(a) C4 (b) E4

(c)C2 (d)G16

PART-B

21. The wave function of the particle at a certain instant is given as $\psi(x) = A\exp\left(-\frac{x^2}{a^2} + ikx\right)$. If P_1 and P_2 denote the probabilities of finding the particle in the range *a* to *a* + *da* and 2*a* to 2*a* + *da* respectively the Ratio P_1 / P_2 is

(a) e^2 (b) e^3

(c) e^6 (d) e^8

22. The value of the integral

$$\int_{-\infty}^{\infty} e^x \sin\left(\frac{\pi x}{3}\right) \delta(4x^2 - 1)\, dx$$

Where $\delta(x)$ represents Dirac Delta distribution is

(a) $\frac{1}{2}\sin h\left(\frac{1}{2}\right)$ (b) $\frac{1}{4}\sin h\left(\frac{1}{2}\right)$

(c) $\frac{1}{8}\sin h\left(\frac{1}{2}\right)$ (d) $\frac{1}{16}\sin h\left(\frac{1}{2}\right)$

23. Given $u = e^{-x}(x\sin y - y\cos y)$ is Harmonic. Find $f(z)$.

(a) ize^{-z} (b) ie^{-z}

(c) $-ize^{-z}$ (d) $-ie^{-z}$

24. Consider a 2 × 2 matrix $A = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The eigen values of 4^A are

(a) (6, 14) (b) (10, 10)

(c) (15, 5) (d) (16, 4)

25. Let $H_n(x)$ be the Hermite polynomial satisfying the following generating function relation :

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$$

The value of $H_6(0)$ is

(a) 1680 (b) – 1680

(c) 120 (d) – 120

26. Consnider the differential equation $dy/dx + y\tan(x) = \cos(x)$. If $y(0)=0$, $y(\pi/3)$ is .... (up to two decimal places).

27. The coefficient of e^{ikx} in the Fourier expansion fo $u(x) = A\sin^2(\alpha x)$ for $k = -2\alpha$ is

(a) A/4 (b) – A/4

(c) A/2 (d) – A/2

28. The contour integral $\oint \frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and cosed in the lower half-plane by a half circle is equal to .... (up to two decimal places).

29. The inverse Laplace transform of the function $f(s) = \frac{1}{s^3(s^2+1)}$ is

(a) $\frac{t^2}{2} - \cos t - 1$ (b) $\frac{t^2}{2} + \cos t - 1$

(c) $\frac{t}{2} + \cos t$ (d) $\frac{t}{2} - \cos t$

30. The absolute value of the integral

$$\int \frac{5z^3 + 3z^2}{z^2 - 4} dz .$$

Over the circle $|z-1.5| = 1$ in complex plane, is .... (up to two decimal places).

31. The product MN of two Hermitian matrices M and N is anti-Hermitian follows that :

(a) $\{M, N\} = 0$ (b) $[M, N] = 0$

(c) $M^\dagger = N$ (d) $M^\dagger = N^{-1}$

32. The wave function (apart from normalization) of a particle of mass ½ unit, confined in the region [0, a] under the one-dimensional potential V(*x*) is $\psi(x,t)=\sin\left(\frac{\pi x}{a}\right)e^{-i\omega t}$. The potential V(*x*) will be of the form (take $\hbar=1$)

(a) $\omega+\frac{\pi^2}{a^2}$ (b) $\omega-\frac{\pi^2}{a^2}$

(c) $\omega+\frac{2\pi^2}{a^2}$ (d) $\omega-\frac{2\pi^2}{a^2}$

33. A particle is in a state which is superposition of the ground state φ_0 and first excited state φ_1 of a one-dimensional infinite potential well of width L. The state is given as following : $|\psi(x)\rangle=|\varphi_0\rangle+2|\varphi_1\rangle$. The uncertainity in the energy of the particle is the given state, will be

(a) $\frac{3\pi^2\hbar^2}{2mL^2}$ (b) $\frac{3\pi^2\hbar^2}{10mL^2}$

(c) $\frac{3\pi^2\hbar^2}{mL^2}$ (d) $\frac{3\pi^2\hbar^2}{5mL^2}$

34. An electron is confined in the ground state of a one-dimensional harmonic oscillator such that $\Delta x=10^{-10}\,m$. The energy required to excite the electron to the first excited state, will be

(a) 2.17 eV (b) 3.11 eV

(c) 3.79 eV (d) 4.56 eV

35. A particle of mass *m* is confined in the following 1-D potential :

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & for\ x>0\\ \infty & \text{otherwise}\end{cases}$$

Suppose the particle is in the following superposition state $|\psi\rangle=\frac{1}{2}|\varphi_0\rangle+i\frac{\sqrt{3}}{2}|\varphi_1\rangle$, where $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are the normalized eigenfunctions of the ground state and the first excited state respectively. The average energy of the particle, is

(a) $1.25\hbar\omega$ (b) $3\hbar\omega$

(c) $2.5\hbar\omega$ (d) $2\hbar\omega$

36. The Eigen function for the ground state of the H-atom has the from $\psi_1(r) = N_1 e^{-r/a_0}$ and excited state $\psi_2(r) = N_2(1+2\lambda r)e^{-r/2a_0}$ using the orthogonality condition for ψ_1 and ψ_2. The constant λ is

(a) $\lambda = -\frac{1}{2a_0}$ (b) $\lambda = -\frac{1}{4a_0}$

(c) $\lambda = -\frac{1}{3a_0}$ (d) None of these

37. The ground state energy E of an electron of mass me is a one-dimensional box of size L is given by $E = \frac{\pi^2\hbar^2}{2mL^2}$, if L is estimated with an uncertainty of 5% then the uncertainty in the estimation of E is.

(a) 10% (b) 25%

(c) 0.04% (d) 0.1%

38. A particle of mass *m* is confined to a one dimension region $0 \le x \le a$ at t = 0 its normalized wave function is given by $\psi(x,t=0) = \sqrt{\frac{8}{5a}}\left[1+\cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$ what is average energy of the system.

(a) $\frac{4\pi^2\hbar^2}{5ma^2}$ (b) $\frac{16\pi^2\hbar^2}{5ma^2}$

(c) $\frac{8\pi^2\hbar^2}{2ma^2}$ (d) $\frac{10\pi^2\hbar^2}{2ma^2}$

39. Find the momentum space wave function $\phi(P,t)$ for a particle which has position space wave function

$$\psi(x,t) = Ae^{-ax^2}e^{-iE_0t/\hbar}$$

(a) $\frac{A}{\sqrt{2a\hbar}}e^{-4P^2/a\hbar^2}e^{-iE_0t/\hbar}$ (b) $\frac{A}{\sqrt{2a\hbar}}e^{-P^2/4a\hbar^2}e^{-iE_0t/\hbar}$

(c) $\frac{\sqrt{2}A}{\sqrt{a\hbar}}e^{-4P^2/a\hbar^2}e^{-iE_0t/\hbar}$ (d) $\frac{\sqrt{2}A}{\sqrt{a\hbar}}e^{-P^2/4a\hbar^2}e^{-iE_0t/\hbar}$

40. Consider operator $\hat{A} = \hat{x}\frac{d}{dx}$ and $\hat{B} = \hat{A}^{\dagger}$. The commutator $\left[\hat{B}, \sin x\right]$ is

(a) $-x\cos x$ (b) $-\sin x$

(c) $-x\sin x$ (d) $x\cos x$

41. If $\hat{A}$ and $\hat{B}$ are Hermitian, which of the following is not Hermitian?

(a) $i(\hat{A}\hat{B} - \hat{B}\hat{A})$ (b) $i(\hat{A} - \hat{A}^{+})$

(c) $(\hat{A}\hat{B} - \hat{B}\hat{A})$ (d) $(\hat{A}\hat{B} + \hat{B}\hat{A})$

42. Consider a harmonic oscillator in the superposition state

$$\psi(x,0) = \frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$$

Then find the expectation value <x> in time dependent state $\psi(x,t)$

(a) $\sqrt{\frac{\hbar}{2m\omega}}\sin(\omega t)$ (b) $\sqrt{\frac{\hbar}{2m\omega}}\cos(\omega t)$

(c) $\sqrt{\frac{\hbar}{2m\omega}}\sin(2\omega t)$ (d) $\sqrt{\frac{\hbar}{2m\omega}}cos(2\omega t)$

43. Consider a particle of mass M constrained to move on a (frictionless) ring of radius a with its centre at O in $x-y$ plane, then the Hamiltonian of the system

(a) $\frac{\hat{L}_x^2}{2I}$ (b) $\frac{\hat{L}_y^2}{2I}$

(c) $\frac{\hat{L}_z^2}{2I}$ (d) None

Where I = moment of inertia of the particle with respect to the centre O.

44. Imagine a diatomic molecule (at low temperature) as a rigid rotator, two particles (atoms) each of mass M separated by a weightless reigid rod of length 2*a* . the mid point of the rotator is fixed. Find the energy eigenvalues

(a) $E_l = \frac{\hbar^2(l+1)(l+2)}{2I}$ (b) $E_l = \frac{\hbar^2 l(l+1)}{2I}$

(c) $E_l = \frac{\hbar^2(l+1)(l+2)}{I}$ (d) $E_l = \frac{\hbar^2 l(l+1)}{I}$

45. At a given instant of time, a rigid rotator is in the state

$$Y(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\phi$$

If L_z is measured on state $Y(\theta,\phi)$, then what is the probability of finding the value of measurement is $+\hbar$?

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

PART-C

46. At time $t=0$, a free particle is in the (normalized) state $\psi(r,0) = A\sin(5\pi x)e^{i(6\pi y} + 4\pi z)$

(a) $\frac{75\pi^2\hbar^2}{2m}$ (b) $\frac{79\pi^2\hbar^2}{2m}$

(c) $\frac{77\pi^2\hbar^2}{2m}$ (d) $\frac{71\pi^2\hbar^2}{2m}$

47. An electron in hydrogen atom is in superposition state described by the wave function

$$\psi(r) = A[4\psi_{100}(r) - 2\psi_{211}(r) + \sqrt{6}\psi_{210}(r) - \sqrt{10}\psi_{21-1}(r)]$$

What is the expectation value of $\hat{L}^2$?

(a) $\frac{7}{9}\hbar^2$ (b) $\frac{11}{9}h^2$

(c) $\frac{10}{9}\hbar^2$ (d) $\frac{15}{9}\hbar^2$

48. Consider an electron at rest in a constant uniform magnetic field in *x* – direction B = B_0 e_x. The corresponding Hamiltonian is :

$$\hat{H}_0 = -\mu \cdot B = -\mu_x B_0 = \mu_B B_0 \hat{\sigma}_x$$

What are the eigenvalues and eigenstates of $\hat{H}_0$?

(a) $\mu_B B_0$ and $-\mu_B B_0$ corresponding to $\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

(b) $\mu_B B_0$ and $-\mu_B B_0$ corresponding to $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ respectively

(c) $\mu_B B_0$ and $\mu_B B_0$ corresponding to $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ respectively

(d) $\mu_B B_0$ and $\mu_B B_0$ corresponding to $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$ respectively

49. Express the spinor $\begin{bmatrix}C_1\\C_2\end{bmatrix}$ in a co-ordinate system obtained by rotation about X-axis through an angle 60°.

(a) $C_1' = \frac{\sqrt{3}}{2}C_1 + \frac{i}{2}C_2,\ C_2' = \frac{-i}{2}C_1 + \frac{\sqrt{3}}{2}C_2$

(b) $C_1' = \frac{-\sqrt{3}}{2}C_1 + \frac{i}{2}C_2,\ C_2' = \frac{i}{2}C_1 - \frac{\sqrt{3}}{2}C_2$

(c) $C_1' = \frac{\sqrt{3}}{2}C_1 - \frac{i}{2}C_2,\ C_2' = \frac{-i}{2}C_1 + \frac{\sqrt{3}}{2}C_2$

(d) $C_1' = \frac{\sqrt{3}}{2}C_1 - \frac{i}{2}C_2,\ C_2' = \frac{i}{2}C_1 - \frac{\sqrt{3}}{2}C_2$

50. Suppose a 2×2 matrix $X = \begin{bmatrix}X_{11} & X_{12}\\X_{21} & X_{22}\end{bmatrix}$ is written as

$$X = b_0 I + \hat{\sigma}\cdot b$$

Where $b_0,\ b_k (k=1,2,3)$ are numbers. Find the value of $tr(X)$ and $tr(\hat{\sigma}_k X)$

(a) $b_0,\ b_k$

(b) $2b_0,\ 2b_k$

(c) $2b_0, b_k$ (d) $b_0, 2b_k$

51. Consider a two level system whose Hamiltonian operator is given as

$$\hat{H} = a\left[|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right]$$

Here number a is having dimension of energy. Find the energy eigenstates in terms of states $|1\rangle \& |2\rangle$

(a) $|\psi_1\rangle = A|1\rangle + A\left(\sqrt{2}-1\right)|2\rangle, |\psi_2\rangle = -B|1\rangle + B\left(\sqrt{2}+1\right)|2\rangle,$

(b) $|\psi_1\rangle = A|1\rangle - A\left(\sqrt{2}+1\right)|2\rangle, |\psi_2\rangle = -B|1\rangle + B\left(\sqrt{2}-1\right)|2\rangle$

(c) $|\psi_1\rangle = A|1\rangle + A\left(\sqrt{2}+1\right)|2\rangle, |\psi_2\rangle = B|1\rangle + B\left(\sqrt{2}-1\right)|2\rangle$

(d) $|\psi_1\rangle = -A|1\rangle + A\left(\sqrt{2}-1\right)|2\rangle, |\psi_2\rangle = B|1\rangle + B\left(\sqrt{2}+1\right)|2\rangle$

52. A two level unperturbed system is described the Hamiltonian

$$\hat{H}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

Now a small perturbation is switched on, which may be represented by

$$V' = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

Then find the first order correction in energy eigenvalues

(a) a, b (b) b, a

(c) c, b (d) a, c

53. A one-dimensional harmonic oscillator is in the state $|\psi\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}}|n\rangle$, where $|n\rangle$ is the normalized energy eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. Let the expectation value of the Hamiltonian in the state $|\psi\rangle$ be expressed as $\frac{1}{2} \propto \hbar\omega$, then the value of $\propto$ is

(a) 5 (b) 3

(c) $n+1$ (d) $2n+1$

54. If the electric field in *z*-direction is switched on at $t=-\infty$ and varies with time as

$$E(t)=\frac{E_0}{\pi}\frac{\tau}{t^2+\tau^2}$$

Find the probability of excitation of hydrogen atom from its ground state to 2P state for $t(>>\tau)$.

(a) $\frac{A^2E_0^2}{\hbar^2}e^{-2\omega\tau}$ (b) $\frac{A^2E_0^2}{\hbar^2}e^{-\omega\tau}$

(c) $\frac{A^2E_0^2}{\hbar^2}e^{-\omega\tau/2}$ (d) $\frac{A^2E_0^2}{\hbar^2}e^{-\omega\tau/4}$

55. Find out the ratio of the number of stimulated emissions to that of the spontaneous emissions in thermal equilibrium at temperature T = 300 K for visible region $\lambda\sim 5000\AA$

(a) 0 (b) 5.7

(c) 20.3 (d) 9.3

56. Find the value of commutation relation

$$\left[\hat{L}^2,\left[\hat{L}^2,r\right]\right]$$

(a) $\hbar^2(r\hat{L}^2+\hat{L}r^2)$ (b) $2\hbar^2(r\hat{L}^2+\hat{L}^2r)$

(c) $\hbar^2(r^2\hat{L}+\hat{L}r^2)$ (d) $2\hbar^2(r^2\hat{L}+\hat{L}r^2)$

57. Consider the two spin $\frac{1}{2}$ particles in the symmetric state is given by

$$|\phi(1,2)\rangle=\frac{1}{\sqrt{2}}\left[|\alpha_z\rangle_1|B_z\rangle_2+|B_z\rangle_1|\alpha_z\rangle_2\right]$$

Then find the eigen value of $\hat{S}^2$

(a) $2\hbar^2$ (b) $\hbar^2$

(c) $\frac{+\hbar^2}{2}$ (d) $\frac{+\hbar^2}{4}$

58. If $\vec{\sigma}$ is Pauli's spin operator for a spin $-\frac{1}{2}$ particle and $\hat{n}$ is unit vector, then the commutator $\left[\vec{\sigma}\cdot\hat{n}, \vec{\sigma}\right]$ is

(a) $2i\vec{\sigma}\cdot(\vec{\sigma}\times\hat{n})$ (b) $i(2\hat{n}(\vec{\sigma}\cdot\hat{n})-\vec{\sigma})$

(c) $2i(\vec{\sigma}\times\hat{n})$ (d) $i\vec{\sigma}\cdot(\vec{\sigma}\times\hat{n})-i\vec{\sigma}$

59. A polar representation of the creation and annihilation operators for a simple harmonic oscillator can be introduced as $a=\sqrt{N+1}e^{i\phi}$ and $a^{\dagger} \; = e^{-i\phi}\sqrt{N+1}$

The operators N and ϕ are assumed to be Hermitian. Given $\left[a,\, a^{\dagger}\right]=1,$ the value of $\left[\cos\phi,\, N\right]$ is

(a) $\cos\phi$ (b) $-e^{-i\phi}$

(c) $e^{i\phi}$ (d) $i\sin\phi$

60. A two-state quantum system has energy eigenvalues $\pm\in$ corresponding to the normalized states $\left|\psi_{\pm}\right\rangle$. At time $t=0$, the system is in quantum state $\frac{1}{\sqrt{2}}\left[\left|\psi_{+}\right\rangle+\left|\psi_{-}\right\rangle\right]$. The probability that the system will be in the same state at $t=h/(6\in)$ is .... (up to two decimal places).

61. Let A and B represent two types of non-interacting sping -1/2 particles. If 3 particles of type A and 4 particles of type B are in a simple harmonic oscillator potential characterized by ω, the ground state energy of the system is (M5)

(a) $\frac{9}{2}\hbar\omega$ (b) $\frac{11}{2}\hbar\omega$

(c) $\frac{13}{2}\hbar\omega$ (d) $\frac{15}{2}\hbar\omega$

62. Electrons in a given system of hydrogen atoms are described by the wave function

$$\psi(\gamma,\theta,\varphi)=0.8\Psi_{100}+0.6\, e^{i\pi/3}\Psi_{311}$$

Where the $\Psi_{n\ell m}$ denote normalized energy eigenstates. If $(\hat{L}_x,\hat{L}_y,\,\hat{L}_z)$ are the components of the orbital angular momentum operator, the expectation value of $\hat{L}_x^2$ in this system is

(a) $1.5\hbar^2$ (b) $0.36\hbar^2$

(c) $0.18\hbar^2$ (d) Zero

63. A quantum mechanical system which has stationary states $|1\rangle, |2\rangle$ and $|3\rangle$, corresponding to energy levels $0eV, 1eV$ and $2eV$ respectively, is perturbed by a potential of the form

$$\hat{V} = \varepsilon|1\rangle\langle 3| + \varepsilon|3\rangle\langle 1|$$

Where, in eV, $0 < \varepsilon << 1$.

The new ground state, correct to order ε, is approximately.

(a) $\left(1-\frac{\varepsilon}{2}\right)|1\rangle + \frac{\varepsilon}{2}|3\rangle$ (b) $|1\rangle + \frac{\varepsilon}{2}|2\rangle - \varepsilon|3\rangle$

(c) $|1\rangle + \frac{\varepsilon}{2}|3\rangle$ (d) $|1\rangle - \frac{\varepsilon}{2}|3\rangle$

64. A quantum particle of mass *m* is moving on a horizontal circular path of radius *a* . the particle is prepared in a quantum state described by the wavefunction.

$$\psi = \sqrt{\frac{4}{3\pi}}\cos^2\phi$$

ϕ being the azimuthal angle. If a measurement of the z-component of orbital angular momentum of the particle is carried out, the possible outcomes and the corresponding probabilities are

(a) $L_z = 0, \pm\hbar, \pm 2\hbar$ with $P(0) = \frac{1}{5}$, $P(\pm\hbar) = \frac{1}{5}$ and $P(\pm 2\hbar) = \frac{1}{5}$

(b) $L_z = 0$ with $P(0) = 1$

(c) $L_z = 0, \pm\hbar$ with $P(0) = \frac{1}{3}$ and $P(\pm\hbar) = \frac{1}{3}$

(d) $L_z = 0, \pm 2\hbar$ with $P(0) = \frac{2}{3}$ and $P(\pm 2\hbar) = \frac{1}{6}$

65. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(x)$ is NOT CORRECT?

(a) $F(t) = \delta(t)$, $f(s) = 1$, (Singularity at + 0)

(b) $F(t)=1,\ f(s)=\frac{1}{s},\ (s>0)$

(c) $F(t)=\sin kt,\ f(x)=\frac{s}{s^2+k^2},\ (s>0)$

(d) $F(t)=te^{kt},\ f(x)=\frac{1}{(s-k)^2},(s>k,s>0)$

66. The Fourier transform $\int_{-\infty}^{\infty} dx\, f(x)e^{ikx}$ of the function $f(x)=\frac{1}{x^2+2}$ is

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$

(b) $\sqrt{2}\pi\, e^{-\sqrt{2}k}$

(c) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2}k}$

(d) $\frac{\pi}{\sqrt{2}}e^{-\sqrt{2}|k|}$

67. For a fixed positive integer $n\geq 3$, let A be the $n\times n$ matrix defined $A=I-\frac{1}{n}J,$ where I is the identity matrix J is the $n\times n$ matrix with all entries equal to 1. Which of the following statements Is NOT true?

(a) $A^k=A$ for every positive integer *k*

(b) Trade (A) = *n* – 1

(c) Rank (A) + Rank (1 – A) = *n*

(d) A is invertible

68. Select the incorrect statement. If λ is an eigen value of A then

(a) λ^{100} is an eigen value of A^{100}

(b) $|A|/\lambda$ is an eigen value of adjA

(c) $(\lambda+2)^5$ is an eigen value of $(A+2I)^5$

(d) $1/\lambda$ is an eigen value of A adj A

69. The symmetry elements of a square ABCD form a group, $G=\{C_4, C_4^2, C_4^3, C_4^4, \sigma_x, \sigma_y, \sigma_{AC}, \sigma_{BD}\}$ under multiplication where

C_4, C_4^2, C_4^3, C_4^4 are the rotational symmetry elements and $\sigma_x, \sigma_y, \sigma_{AC}, \sigma_{BD}$ are the reflection symmetry elements. The operation $C_4^2\sigma_x$ is equivalent to

(a) σ_{AC} (b) σ_{BD}

(c) σ_y (d) C_4

70. The function $f_a(x)$ defined as unity for $0 < x < a$ and zero otherwise. If its Laplace transforms is $\tilde{f}_a(s)$, then the Laplace transform of $xf_a(x)$ is

(a) $\frac{1}{s^2}\left[1-(1+as)e^{-sa}\right]$ (b) $\left[1-(1+as)e^{-sa}\right]$

(c) $\frac{1}{s^2}\left[1-ase^{-sa}\right]$ (d) $\left[1-ase^{-sa}\right]$

71. A set of N vectors X_1, X_2 , ... X_n satisfy the eigenvalue equation for an operator A with scalar eigenvalues $\lambda_1, \lambda_2, ...\lambda_n$ $(i.e.\ AX_k = \lambda_K X_K)$. The linear combination vector $X = \sum_{k=1}^{N} C_k X_K$ where C_k's are non-zero scalar coefficient.

(a) X is not an eigen vector of A

(b) X is an eigenvector of A only if λ_k's are all distinct

(c) X is an eigenvector of A only if the λ_k's are equal

(d) X is an eigenvector of A if C_k's are equal

72. Let $1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$ are eigenvalues of a 3×3 matrix A. If A can be expanded as power series expansion of exponential as $e^A = \sum_{n=0}^{\infty}\frac{A^n}{n!}$, then the value of det (e^A) is

(a) $e-\frac{1}{e}$ (b) e^2-e

(c) 1 (d) 0

73. Taylor expansion of $f(z)=\dfrac{1+2z^2}{z^3+z^5}$ around $z=0$ is given by

(a) $\left(\frac{1}{z^3}+\frac{1}{z}\right)+\sum_{n=0}^{\infty} z^{2n+1}$

(b) $\left(\frac{1}{z^3}+\frac{1}{z}\right)-\sum_{n=0}^{\infty} (-1)^n z^{2n+1}$

(c) $\left(\frac{1}{z^3}-\frac{1}{z}\right)-\sum_{n=0}^{\infty} z^{2n+1}$

(d) $\left(\frac{1}{z^3}-\frac{1}{z}\right)-\sum_{n=0}^{\infty} (-1) z^{2n+1}$

74. Let A, B, C, D be $n\times n$ matrices over R. Assume that AB^T and CD^T are symmetric and $AD^T-BC^T=I_n$ where T denotes transpote

(a) $A^TD-C^TB=0_n$

(b) $A^TD-CB^T=0_n$

(c) $A^TD-C^TB=I_n$

(d) none of these

75. Consider the following initial value problem :

$$y^n+2y'+10y=6\delta(t-2)-3\delta(t-3),\ y(0)=0,\ y'(0)=0$$

Where $\delta(t-a)$ is the dirac delta function. The Laplace transform of the solution $y(t)$ of the differential equation, will be

(a) $\dfrac{6e^{-2s}-3e^{-3x}}{s^2+2S+10}$

(b) $\dfrac{6e^{2s}-3e^{3s}}{s^2+2S+10}$

(c) $\dfrac{6e^{-2s}-3e^{-3s}}{s^2-2S-10}$

(d) None of these