CSIR-NET Full length TEST PAPER

PHYSICS By A. Singh Sir
IIT Roorkee
(CSIR-JRF, GATE (12th, 19th AIR), BARC, JEST)

ATS-2

(Full Length-Q.M. & ED.)

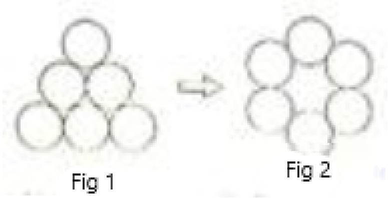
PART-A

1 .Of the following, which is the odd one out ?

(a) CONE (b) TORUS

(c) ELLIPSE (d) SPHERE

2. What is the minimum number of moves required to transform figure 1 to figure 2 ? A move is defined as removing a coin and placing it such that it touches two other coins in its new position.



(a)4 (b)1

(c)2 (d)3

3. What comes next in the sequence ?

(a)  (b) 

(c)  (d) 

4. Arrange the words given below in a meaningful sequence.

1.Presentation 2.Recommendation 3. Arrival 4.Discussion 5.Introduction

(a) 5,3,4,2,1 (b) 3,5,4,2,1

(c) 3,5,1,4,2 (d) 5,3,2,4,1

5. What was the day on 15th august 1947 ?

(a)friday (b)saturday
(c)Wednesday (d)Thursday

6. Today is Monday. After 61 days, it will be :

(a)Tuesday (b)Monday
(c)Sunday (d)Saturday

7. A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.

(a) 360, 160, 200 (b) 160, 360, 200

(c) 200, 360,160 (d) 200,160,300

8. In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got, was :

(a)2500 (b)2700
(c)2900 (d)3100

9. When 96 is multiplied to 25% of a number the result thus obtained is obtained 75% of 9408. What is the number?

(a) 294 (b) 300

(c)291 (d)295

10. Twenty years ago suresh was four times older than raman. After 20 years, the suresh will be twice older than raman. The present age of raman is:

(a) 45 years (b) 35 years

(c) 50 years (d) 40 years

11. An amount is given at an interest of 20% per annum. What is the amount, if the difference between CI and SI for 3 years is Rs. 1049.6?

(a)8300 (b)8100
(c)8500 (d)8200

12. The area of two circles is 3850cm^2 and 1386cm^2 respectively. What is the difference between the circumference of the larger circle and the smaller circle?

(a)90 (b)88
(c)87 (d)85

13. In a 100 ltr mixture of paint and oil, paint is only 70%. The shopkeeper sold 20 ltr of this mixture and then he added 14 ltr of paint and 16 ltr of oil in the remaining mixture. What is the percentage of oil in the final mixture?

(a) 30.10% (b) 33.21%

(c) 36.36% (d) 31.36%

14. How many such pairs of digits are there in the number 421579368 each of which has as many digits between them in the number as when they are arranged in ascending order?

(a)One (b)Two
(c)Three (d)None

15. If the price of a book is first decreased by 25% and then increased by 20%, then the net change in the price will be :

(a)10 (b)20
(c)30 (d)40

16. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

(a)1/2 (b)3/5
(c)9/20 (d)8/15

17. A grocer has a sale of Rs 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs, 6500 ?

(a) 4991 (b) 5467
(c) 5987 (d) 6453

18. An accurate clock shows 8 o'clock in the morning. Through how may degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?

(a)360 (b)180
(c)90 (d)60

19. Which of the following best approximates sin (0.5°) ?

(a)0.5 (b) 0.5×π90
(c) 0.5×π180 (d) 0.5×π360

20. N is a four digit number. If the leftmost digit is removed, the resulting three digit number is 1/9th Of N. How many such N are possible ?

(a)10 (b)9
(c)8 (d)7

(PART-B)

21. Calculate the electric field and the potential at the center of a circle carrying a line charge density $\lambda = \lambda_0 \cos^2\theta$

(a) $E = -\dfrac{\lambda_0}{4\pi\varepsilon_0}, \ \phi = \dfrac{\lambda_0}{4\varepsilon_0}$

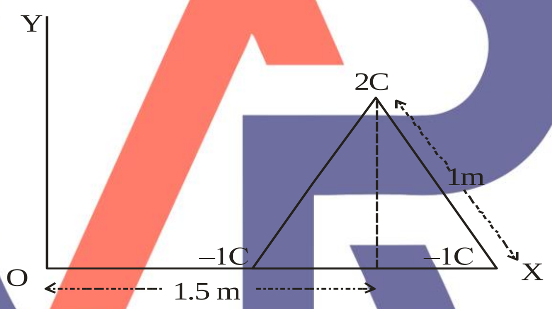
(b) $E = -\dfrac{\lambda_0}{4\pi\varepsilon_0}\hat{i}, \ \phi = \dfrac{\lambda_0}{2\varepsilon_0}$

(c) $E = 0, \ \phi = \dfrac{\lambda_0}{4\varepsilon_0}$

(d) $E = 0, \ \phi = \dfrac{\lambda_0}{2\varepsilon_0}$

22. A uniform volume charge density is placed inside a conductor (with resistivity 10^{-2} Ωm). The charge density becomes 1/(2.718) of its original value after time ………………. femto seconds. (up to two decimal places) ($(\varepsilon_0 = 8.854 \times 10^{-12} F/m)$

23. Three charges (2C, –1 C, –1 C) are placed at the vertices of an equilateral triangle of side 1m as shown in the figure. The component of the electric dipole moment about the marked origin along the $\hat{y}$ direction is _________ C m.



24. A monochromatic plane wave in free space with electric field amplitude of 1 V/m is normally incident on a fully reflecting mirror. The pressure exerted on the mirror is …………. × 10^{-12} Pa. (up to two decimal places) (ε_0 = 8.854 × 10^{-12} F/m).

25. Light is incident from a medium of refractive index n = 1.5 onto vacumm. The smallest angle of incidence for which the light is not transmitted into vacumm is ……………. degree. (up to two decimal places).

26. Identical charges q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively.

(a) 0, 0

(b) $\dfrac{q}{4\pi\varepsilon_0 a^2}, \dfrac{q}{4\pi\varepsilon_0 a}$

(c) $\dfrac{q}{4\pi\varepsilon_0 a^2}, \dfrac{5q}{4\pi\varepsilon_0 a}$

(d) $\dfrac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \dfrac{\sqrt{5}q}{4\pi\varepsilon_0 a}$

27. An electromagnetic plane wave is propagating with an intensity I = 1.0×10^5 Wm^{-2} in a medium with $\in = 3\in_0$ and $\mu = \mu_0$. The amplitude of the electric field inside the medium is ……………. $\times 10^3$ Vm^{-1} (up to one decimal place).

$$(\in_0 = 8.85\times10^{-12} C^2N^{-1}m^{-2},\ \mu_0 = 4\pi\times10^{-7} NA^{-2},\ c = 3\times10^8 ms^{-1})$$

28. A constant and uniform magnetic field $\vec{B} = B_0\hat{k}$ pervades all space. Which one of the following is the correct choice for the vector potential in Coulomb gauge?

(a) $-B_0(x+y)\hat{i}$ (b) $B_0(x+y)\hat{j}$

(c) $B_0x\hat{j}$ (d) $-\frac{1}{2}B_0(x\hat{i}-y\hat{j})$

29. A dielectric shell of inner and outer radii R_1 and R_2 has polarization $\vec{P} = \frac{P_0\hat{r}}{r^2}$. Electric potential at the centre of the shell is :

(a) $\frac{P_0}{\varepsilon_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ (b) $\frac{P_0}{\varepsilon_0}(R_1+R_2)$

(c) $\frac{P_0}{\varepsilon_0}\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$ (d) $\frac{P_0}{\varepsilon_0}(R_2-R_1)$

30. An assembly of charge $+q, -q, +q, -q$, ………. are placed at distance x = 1 m, x = 2m, x = 4m, x = 8 m, …. From the origin, in a plane. The potential at x = 0 due to the charges would be

(a) $-\frac{q}{4\pi\varepsilon_0}$ (b) $-\frac{q}{6\pi\varepsilon_0}$

(c) $\frac{q}{6\pi\varepsilon_0}$ (d) $\frac{q}{4\pi\varepsilon_0}$

31. The electric field in a conducting medium is given by $\vec{E}(x, y) = 5e^{-2x}e^{i(3x-\omega t)}\hat{j}$

The propagation vector and phase difference between $\vec{E}$ and $\vec{B}$ are

(a) $2-2i,\ \phi = \tan^{-1}\left(-\frac{2}{3}\right)$ (b) $2+3i,\ \phi = \tan^{-1}\left(\frac{3}{2}\right)$

(c) $2-3i,\ \phi=\tan^{-1}\left(-\frac{3}{2}\right)$ (d) $3+2i,\ \phi=\tan^{-1}\left(\frac{2}{3}\right)$

32. A long cylinder of radius R carries a current I uniformly distributed on cross-section value of $\vec{\nabla}\times\vec{B}$ is

(a) $\frac{\mu_0 I}{2\pi R}$ inside, zero outside (b) $\frac{\mu_0 I}{\pi R^2}$ inside, zero outside

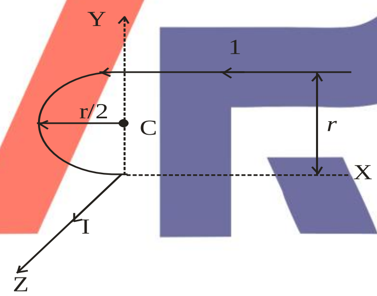
(c) $\mu_0 I$ inside, zero outside (d) zero inside, zero outside.

33. A plane e.m. wave travelling along + z-direction has its electric vectors given by $E_x = 3\cos\omega t$ and $E_y = 3\cos(\omega t + 90°)$. The wave is

(a) Linearly polarized (b) Right circularly polarized

(c) Left circularly polarized (d) Elliptically polarized

34. An infinite long wire as shown in figure carrying a current I with a semicircular loop on xy-plane and two straight parts, one parallel to x-axis and another coinciding with +z-axis. What is the magnetic field induction at the centre C of the loop.



(a) $\frac{\mu_0 I}{2\pi r}\left[(1+\pi)\hat{k}+\hat{i}\right]$ (b) $\frac{\mu_0 I}{2\pi r}\left[(1+\pi)\hat{k}-\hat{i}\right]$

(c) $\frac{\mu_0 I}{4\pi r}\left[(1+\pi)\hat{k}-\hat{i}\right]$ (d) $\frac{\mu_0 I}{2\pi r}\left[(1-\pi)\hat{k}-\hat{i}\right]$

35. The magnetic field induction at the centre 0 of the two concentric coils each carrying equal current I and each of radius r inclined at 90° is

(a) $\frac{\mu_0 I}{r}$ (b) $\frac{\sqrt{2}\mu_0 I}{2r}$

(c) 0 (d) $\frac{\mu_0 I}{2r}$

36. let $|\alpha,\rangle|\beta\rangle, |\gamma\rangle$ represents three ortho-normalized wave functions and two state is defined :

$$|\psi\rangle = c_1\left[|\alpha\rangle + 2|\beta\rangle + (1+i)|\gamma\rangle\right]$$

$$|\phi\rangle = c_2\left[|\alpha\rangle - i|\beta\rangle + |\gamma\rangle\right]$$

The real, positive values of c_1 and c_2 for which $|\psi\rangle \text{and} |\phi\rangle$ are normalized are

(a) $\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{7}}$

(c) $\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{7}}$ (d) $\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{3}}$

37. Consider an electron in Hydrogen atom in a state whose unnormalized wave function is given by :

$R(r) = Ar\left(1 - \frac{r}{6a_0}\right)e^{-\frac{r}{3a_0}}$, where a_0 is first bohr radius.

The expectation value of square of orbital angular momentum is :

(a) $6\hbar^2$ (b) $3\hbar^2$

(c) $2\hbar^2$ (d) 0

38. If H is the Hamiltonian for a free particle with mass *m*, the commutator $[x,[x, H]]$ is

(a) $\hbar^2 / m$ (b) $-\hbar^2 / m$

(c) $-\hbar^2 / (2m)$ (d) $\hbar^2 / (2m)$

39. The normalized wave function of a particle can be written as

$$\psi(x) = N\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{7}}\right)^n \varphi_n(x)$$

Where $\varphi_n(x)$ are the normalized energy eigenfunctions of a given Hamiltonian. The value of N is

(a) $\sqrt{1/7}$ (b) $\sqrt{6/7}$

(c) $\sqrt{3/7}$ (d) $\sqrt{(6-2\sqrt{7})/7}$

40. The operator *a* (defined below) when operating on a harmonic energy eigenstate ψ_n with energy E_n, produces another energy eigenstate whose energy is $E_n - \hbar\omega_0$. Which of the following is true?

$$a = \sqrt{\frac{m\omega_0}{2\hbar}}\left(x + i\frac{p}{m\omega_0}\right)$$

I. *a* commutes with the Hamiltonian.

II. *a* is a Hermitian operator and therefore an observable.

III. The adjoint operator $a^\dagger \neq a$

(a) I only (b) II only

(c) III only (d) I and II only

41. A very simple square well potential is given :

$$V(x) = \begin{cases} \infty & x > 0 \\ 0 & 0 < x < L \\ \infty & x > L \end{cases}$$

Density of state of this quantum system depend on

(a) $L E^{-1/2}$ (b) $L^2 E^{-1/2}$

(c) $L^3 E^{-1/2}$ (d) $L^{-1} E^{-1/2}$

42. Consider an operator $\hat{A}$ defined by, $\hat{A} = \left[x, \hat{p}_x^2\right]$. The eigenvalues of the operator will be.

(a) real (b) purely imaginary

(c) of unit modulus (d) unity

43. At t = 0, the wavefunction of another – wise free particle confined between two infinite walls at x = 0 and x = L is $\psi(x, t=0) = \sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)$. Its wave function at a later time $t = \frac{mL^2}{4\pi\hbar}$ is

(a) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)e^{i\pi/6}$

(b) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} + \sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

(c) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$

(d) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} + \sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$

44. A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}}[\psi_\alpha(x_1)\psi_\beta(x_2) + \psi_\beta(x_1)\psi_\alpha(x_2)]$$

Where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particle might be

(a) Electrons

(b) Positrons

(c) Protons

(d) Deuterons

45. The value of the commutator, $\left[\hat{x} + \hat{p}_x^2, \hat{p}_x + \hat{x}^2\right]$ is equal to

(a) $i\hbar(1 - 4\hat{p}_x\hat{x})$

(b) $i\hbar(1 - 4\hat{x}\hat{p}_x)$

(c) $i\hbar(1 - 2\hat{p}_x\hat{x} - 2\hat{x}\hat{p}_x)$

(d) $i\hbar(1 + 2\hat{p}_x\hat{x} + 2\hat{x}\hat{p}_x)$

(PART-C)

46. The potential varies in a region of space as $\phi = \phi_0 e^{-bx^2}$ where ϕ_0 and b are constants. The charge density in this region is :

(a) $2b\varepsilon_0 x\phi$

(b) zero

(c) $2b\varepsilon_0\phi(1 + 2bx^2)$

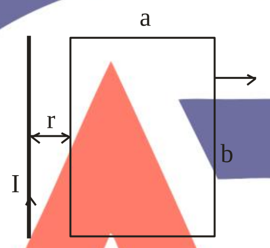
(d) $2b\varepsilon_0\phi(1 - 2bx^2)$

47. A dielectric spherical alyer of radius *a* and *b* (*a* < *b*) carries a polarization $\vec{p} = \frac{k}{r^2}\hat{r}$ where *k* is a constant and *r* is the measured from the centre of the sphere. The bound surface and volume charge densities are respectively given by

(a) 0 and $\frac{+k}{a^2}$ (b) $\frac{-k}{a^2}$ and 4π ka

(c) $\frac{-k}{a^2}$ and 0 (d) $\frac{-k}{a^2}$ and $-4\pi\, ka$

48. A rectangular loop with side *a* and *b* and a long straight wire carrying current *I* are located in the same plane as shown in figure. The loop moves to the right with a constant velocity *v*. The mutual induction between the straight wire and the loop as a function of distance *r* is :



(a) $\frac{\mu_0 b}{2\pi}\ell n\left(\frac{r+b}{r}\right)$ (b) $\frac{\mu_0 a}{2\pi}\ell n\left(\frac{r+a}{r}\right)$

(c) $\frac{\mu_0 b}{2\pi}\ell n\left(\frac{r+a}{r}\right)$ (d) $\frac{\mu_0 a}{2\pi}\ell n\left(\frac{r+b}{r}\right)$

49. A magnetic field $\vec{B} = kz\hat{x}$ exists in space. Force on a square loop (side *a*), lying in *yz* plane and centred at origin and carrying current *I* is.

(a) $\frac{Ika^2}{2}$ (b) $2Ika^2$

(c) Ika^2 (d) zero

50. A large sheet carrying a current density $\vec{k} = k\hat{y}$ lies in *y-z* plane. Magnetic field due to the sheet is

(a) $\frac{\mu_0 k}{2}\hat{x},\ x<0,\ \frac{-\mu_0 k}{2}\hat{x},\ x>0$ (b) $-\frac{\mu_0 k}{2}\hat{z},\ x<0,\ \frac{\mu_0 k}{2}\hat{z},\ x>0$

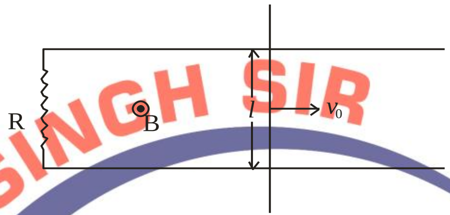
(c) $\frac{\mu_0 k}{2}\hat{z},\ x<0, -\frac{\mu_0 k}{2}\hat{z},\ x>0$ (d) $-\frac{\mu_0 k}{2}\hat{z},\ x<0, \frac{\mu_0 k}{2},\ x>0$

51. In some region electric field (E_0) and magnetic field (B_0) both are parallel to each other. A charge particle enter into this region with initial velocity (v_0) perpendicular to the fields and starts moving in helical path. After some time if we switch off the magnetic field particle will move in :

(a) straight line path (b) helical path

(c) parabolic path (d) hyperbolic path

52. In the given figure, if the wire starts moving with speed v_0 at t = 0, its speed after time t will be



(a) $v = v_0 - \frac{B^2\ell^2 t}{mR}$ (b) $v_0 e^{\frac{B^2\ell^2 t}{mR}}$

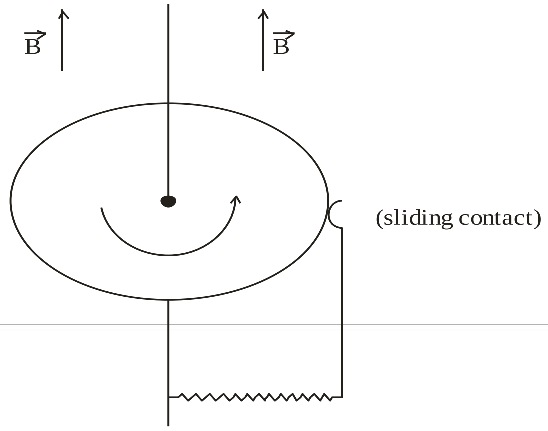
(c) $v_0 e^{-\frac{B^2\ell^2 t}{2mR}}$ (d) $v_0\left(1 - e^{\frac{B^2\ell^2 t}{2mR}}\right)$

53. An electric field is expressed as $\vec{E} = 3\hat{i} + 5\hat{j}$ in the observer fram. What is the electric field in a reference frame moving with velocity $(7\hat{i} + 9\hat{j}) \times 10^7$ m/sec relative to the observers?

(a) $3\hat{i} + 5\hat{j}$ (b) $3.1\hat{i} - 5.1\hat{j}$

(c) $3\hat{i} - 5\hat{j}$ (d) $3.1\hat{i} + 5.1\hat{j}$

54. A metal disk of radius a rotates with angular velocity ω about a vertical axis through a uniform field $\vec{B}$, pointing up. A circuit is made by connecting one end of a resistor to the centre of the disc and the other end to a sliding contact which touches the outer edge of the disk. The current in the resistor is :



(a) $\frac{\omega Ba^2}{R}$

(b) $\frac{\omega Ba^2}{2R}$

(c) $\frac{\omega Ba^2}{3R}$

(d) $\frac{\omega Ba^2}{4R}$

Link 55-56

A metallic square loop of side 'a' meter and resistance 10Ω is placed in 1st quadrant of *y-z* plane with one corner at origin. There is a time varying non-uniform magnetic field $(\vec{B})$ exists in the space.

55. Which one of the following could be a valid magnetic field $(\vec{B})$:

(a) $B_0 e^{-(y+t)}\hat{x}$

(b) $B_0 e^{-(x+t)}\hat{x}$

(c) $B_0 e^{-t}\sin(kx)\hat{x}$

(d) $B_0 e^{-(x+y+z+t)}\hat{x}$

56. The amount of net charge (Q), in coulombs, that flows past any point in the loop from $t=0$ to $t=\infty$ is :

(a) $0.1B_0\, a(1-e^{-a})$

(b) $B_0 a(1-e^{-a})^2$

(c) $0.1B_0 a\left(e^{-a}-1\right)$

(d) $B_0 a(e^{-a}-1)$

Link 57-58

A long straight wire of radius R has a current distribution

$$\vec{J}(r)=\frac{J_0}{r}\cos\left(\frac{\pi r}{2R}\right)\hat{z}$$

Where 'r' is the distance from the axis.

57. Total current (I) in the wire is :

(a) $2J_0R$ (b) $4J_0R$

(c) J_0R (d) 0

58. Magnetic field inside the wire at a distance r < R from the axis is :

(a) $\frac{2\mu_0 J_0 R}{\pi r}\sin\left(\frac{\pi r}{2R}\right)\hat{\phi}$ (b) $\frac{2\mu_0 J_0 R}{\pi r}\sin\left(\frac{\pi r}{2R}\right)\hat{r}$

(c) $\frac{\mu_0 J_0 R}{r^2}\cos\left(\frac{\pi r}{2R}\right)\hat{\phi}$ (d) $2\mu_0 J_0 R\cos\left(\frac{\pi r}{2R}\right)\hat{r}$

59. A solid sphere made of dielectric material has polarization $\vec{P} = P_0\vec{r}$. If R be its radius then dielectric field inside and outside at a distance 'r' r from centre.

(a) $-\frac{P_0}{3\varepsilon_0}\vec{r}, \frac{-P_0}{3\varepsilon_0}\frac{R^2}{r^2}\hat{r}$ (b) $-\frac{P_0}{3\varepsilon_0}\vec{r}$, zero

(c) $-\frac{P_0}{\varepsilon_0}\vec{r}, \frac{-P_0}{\varepsilon_0}\frac{R^2}{r^2}\hat{r}$ (d) $-\frac{P_0}{\varepsilon_0}\vec{r}, \frac{-P_0}{\varepsilon_0}\frac{R^2}{r^2}\hat{r}$

60. Two circular loops of conducting wires having radius R and 2R, carrying current I_0 cos(100□t) and I_0 cos(50□t) respectively. The ratio of power radiated by the loops is :

(a) 1 (b) 4

(c) 32 (d) 16

61. The electric charge distribution that creates an electric potential

$$V(r) = \frac{e^{-\lambda r}}{r}$$

Where λ is a constant.

Will be?

(a) $\frac{-\lambda^2}{r}\varepsilon_0 e^{-\lambda r}$ (b) $4\pi\varepsilon_0\delta^3(r) - \frac{\lambda^2}{r}\varepsilon_0 e^{-\lambda r}$

(c) $e^{-\lambda r}\varepsilon_0\left[4\pi\delta^3(r) + \frac{2\lambda}{r^2} - \frac{\lambda^2}{r}\right]$ (d) $4\pi\varepsilon_0\delta^3(r)$

62. The vector $\vec{A}=\frac{1}{2}\alpha t(x\hat{j}-y\hat{i}, \phi=\frac{1}{4}\alpha(x^2+y^2)$ where 'α' is a constant ant 't' is time. The electric field $(\vec{E})$ and magnetic field $(\vec{B})$ corresponding to these potentials are, respectively.

(a) $\frac{1}{4}\alpha\left[(x+y)\hat{i}+(x-y)\hat{j}\right],\frac{1}{2}\alpha t\hat{k}$ (b) $-\frac{1}{2}\alpha\left[(x-y)\hat{i}+(x+y)\hat{j}\right],\alpha t\hat{k}$

(c) $-\frac{1}{2}\alpha\left[(x\hat{i}+y\hat{j}\right],\alpha t\hat{k}$ (d) $-\frac{1}{4}\alpha\left[(x+y)\hat{i}+(x-y)\hat{j}\right],\alpha t\hat{k}$

63. An electromagnetic wave

$\vec{E}=-20e^{i(4x+3y-5\times10^8 t)}\hat{k}\ V/m$

Is travelling in isotropic linear non-magnetic dielectric medium. The dielectric constant $(\in_r)$ of the medium is :

(a) 3 (b) 9

(c) 5 (d) $\sqrt{3}$

64. The volume current density through a long cylindrical conductor is given to be $\vec{J}=J_0\hat{z}\left(1-\frac{r}{R}\right)$ whre. R is radius of cylinder and 'r' is the distance of some point from the axis of cylinder and J_0 is a constant. The value of r at which magnetic field maximum is :

(a) $\frac{3R}{2}$ (b) $\frac{R}{2}$

(c) $\frac{3R}{4}$ (d) $\frac{R}{4}$

65. Suppose a point charge Q is placed at the centre of the sphere of radius R. The electric flux through the region $\left(\frac{\pi}{6}\le\theta\le\frac{5\pi}{6}\text{and } 0\le\phi\le2\pi\right)$ is

(a) zero (b) $\frac{\sqrt{3}Q}{2\varepsilon_0}$

(c) $\frac{Q}{\sqrt{2}\varepsilon_0}$ (d) $\frac{2Q}{3\varepsilon_0}$

66. The spin-state of an electron in $\hat{S}_z$ basis is given by,

$$|\chi\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix}1+i\\ \sqrt{3}\end{pmatrix}$$

The probability that a measurement of $\hat{S}_y$ will yield the values $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ are, respectively

(a) $\frac{1}{2}+\frac{\sqrt{3}}{5}.\frac{1}{2}-\frac{\sqrt{3}}{5}$ (b) $\frac{1}{2},\frac{1}{2}$

(c) $\frac{1}{2}-\frac{\sqrt{3}}{5},\frac{1}{2}+\frac{\sqrt{3}}{5}$ (d) $\frac{1}{4},\frac{3}{4}$

67. A micro particle of mass *m* moving on a ring of radius a lies in the *xy* plane. The wave function of the particle is given by

$\psi = A\cos^3\phi$ ϕ being the azimuthal angle and A is a constant. If a measurement of the z-component of orbital angular momentum of the particle is carried out, the probability of getting the result $+\hbar$ is

(a) $\frac{1}{20}$ (b) $\frac{9}{20}$

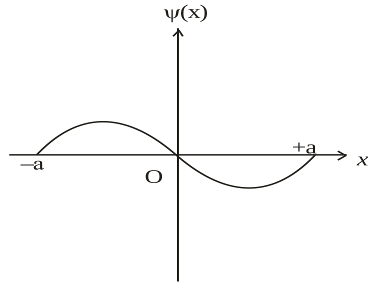
(c) $\frac{3}{10}$ (d) $\frac{1}{10}$

68. A Hermitian operator $\hat{A}$ has only two eigenstates $|\phi_1\rangle$ and $|\phi_2\rangle$ with eigenvalues a_1 and a_2 respectively. If $\langle\hat{A}\rangle$ for a state $|\psi\rangle$ is $\frac{a_1+2a_2}{3}$, the uncertainty in $\hat{A}$ for the state $|\psi\rangle$ will be

(a) $\frac{1}{\sqrt{3}}|a_1-a_2|$ (b) $\frac{\sqrt{2}}{3}|a_1-a_2|$

(c) $\frac{\sqrt{2}}{3}|a_1+a_2|$ (d) $\frac{1}{\sqrt{3}}|a_1+a_2|$

69.



The figure above shows one of the possible energy eigenfunctions $\psi(x)$ for a particle bouncing freely back and forth along the *x*-axis between impenetrable walls located at *x* = −*a* and *x* = + *a*. The potential energy equals zero for |*x*| > *a*. If the energy of the particle is 2 electron volts when it is in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy?

(a) 0 eV

(b) $1/\sqrt{2}eV$

(c) ½ eV

(d) 1 eV

70. Variational parameter of a particle of mass *m* in the potential $V(x)=\frac{\hbar^2\beta}{6m}x^4$, estimated using the normalized trial wave function :

$\Psi(x)=\left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, is β^n, value of *n* is

(a) 1

(b) 1/2

(c) 1/3

(d) 2

71. The Hamiltonian for a spin -1/2 particle at rest is given by H = A (σ_z + σ_x), where σ_x and σ_z are Pauli spin matrices and A is constants. The eigenvalues of this Hamiltonian are :

(a) $\pm A\sqrt{2}$

(b) A

(c) A (doubly degenerate)

(d) $A\left(1\pm\frac{1}{2}\right)$

72. The Coulomb potential $V(r)=-\frac{e^2}{r}$ of a hydrogen atom is perturbed by adding $H'=c(y^2+r^2)$ (where c is a constant) to the Hamiltonian. The first order correction to the ground state energy is in state Ψ_{100}

(a) $-c(a_0)^2$

(b) $4c(a_0)^2$

(c) $-4c(a_0)^2$ (d) none

73. A particle of mass m is moving in a one-dimensional box defined by potential V = $0, 0 \le x < a$ and $V = \infty$ otherwise. Estimate the ground state energy using the trial function $\psi(x) = Ax(a-x), 0 \le x < a$

(a) $\frac{5\hbar^2}{ma^2}$ (b) $\frac{9\hbar^2}{2ma^2}$

(c) $\frac{9\hbar^2}{ma^2}$ (d) $\frac{7\hbar^2}{ma^2}$

74. Consider a particle with mass 'm' in two dimensional square box of length L. There is a weak potential in box $V(x,y) = V_0 L^2 \delta\left(x - \frac{L}{2}\right)\delta\left(y - \frac{L}{4}\right)$ the first order correction to the energy of the first excited state.

(a) 0 (b) V_0

(c) $\frac{V_0}{2}$ (d) $2V_0$

75. $|0\rangle$ and $|1\rangle$ are two orthonormal vectors that constitute the basis of a 2-D Hilbert space. A generic state $|\psi\rangle$ in this space is,

$|\psi\rangle = a|0\rangle + b|1\rangle$ $\left[a^2 + b^2 = 1, a \text{ and } b \text{ are real}\right]$ And an operator, $\hat{A} = |1\rangle\langle 1| + i(|0\rangle\langle 1| - |1\rangle\langle 0|)$. The value of b for which $\langle \hat{A} \rangle$ for the state $|\psi\rangle$ is minimum, will be

(a) $\frac{1}{\sqrt{2}}$ (b) 0

(c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{4}}$